Least squares principle

Contributors

Least squares principle is a widely used method for obtaining the estimates of the parameters in a statistical model based on observed data. Suppose that we have measurements \( Y_1, \ldots, Y_n \) which are noisy versions of known functions \( f_1(\beta), \ldots, f_n(\beta) \) of an unknown parameter \( \beta \). This means, we can write

\[
Y_i = f_i(\beta) + \varepsilon_i, \quad i = 1, \ldots, n
\]

where \( \varepsilon_1, \ldots, \varepsilon_n \) are quantities that measure the departure of the observed measurements from the model, and are typically referred to as \textit{noise}. Then the \textbf{least squares estimate} of \( \beta \) from this model is defined as

\[
\hat{\beta} = \min_{\beta} \sum_{i=1}^n (Y_i - f_i(\beta))^2
\]

The quantity \( f_i(\hat{\beta}) \) is then referred to as the \textit{fitted value} of \( Y_i \), and the difference \( Y_i - f_i(\hat{\beta}) \) is referred to as the corresponding \textit{residual}. It should be noted that \( \hat{\beta} \) may not be unique. Also, even if it is unique it may not be available in a closed mathematical form. Usually, if each \( f_i \) is a smooth function of \( \beta \), one can obtain the estimate \( \hat{\beta} \) by using numerical optimization methods that rely on taking derivatives of the objective function. If the functions \( f_i(\beta) \) are linear functions of \( \beta \), as is the case in a linear regression problem, then one can obtain the estimate \( \hat{\beta} \) in a closed form.

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