6.E: Sampling Distributions (Exercises)

These are homework exercises to accompany the Textmap created for "Introductory Statistics" by Shafer and Zhang.

6.1: The Mean and Standard Deviation of the Sample Mean

Basic

Q6.1.1
Random samples of size \(225\) are drawn from a population with mean \(100\) and standard deviation \(20\). Find the mean and standard deviation of the sample mean.

Q6.1.2
Random samples of size \(64\) are drawn from a population with mean \(32\) and standard deviation \(5\). Find the mean and standard deviation of the sample mean.

Q6.1.3
A population has mean \(75\) and standard deviation \(12\).

a. Random samples of size \(121\) are taken. Find the mean and standard deviation of the sample mean.

b. How would the answers to part (a) change if the size of the samples were \(400\) instead of \(121\)?
Q6.1.4

A population has mean \(5.75\) and standard deviation \(1.02\).

a. Random samples of size \(81\) are taken. Find the mean and standard deviation of the sample mean.

b. How would the answers to part (a) change if the size of the samples were \(25\) instead of \(81\)?

Answers

S6.1.1

\[ \mu_{\bar{X}} = 50, \quad \sigma_{\bar{X}} = 1.33 \]

S6.1.3

a. \[ \mu_{\bar{X}} = 75, \quad \sigma_{\bar{X}} = 1.09 \]

b. \(\mu_{\bar{X}}\) stays the same but \(\sigma_{\bar{X}}\) decreases to \(0.6\)

6.2: The Sampling Distribution of the Sample Mean

Basic

1. A population has mean \(128\) and standard deviation \(22\).
   
   a. Find the mean and standard deviation of \(\overline{X}\) for samples of size \(36\).
   
   b. Find the probability that the mean of a sample of size \(36\) will be within \(10\) units of the population mean, that is, between \(118\) and \(138\).

2. A population has mean \(1,542\) and standard deviation \(246\).
   
   a. Find the mean and standard deviation of \(\overline{X}\) for samples of size \(100\).
   
   b. Find the probability that the mean of a sample of size \(100\) will be within \(100\) units of the population mean, that is, between \(1,442\) and \(1,642\).

3. A population has mean \(73.5\) and standard deviation \(2.5\).
   
   a. Find the mean and standard deviation of \(\overline{X}\) for samples of size \(30\).
   
   b. Find the probability that the mean of a sample of size \(30\) will be less than \(72\).

4. A population has mean \(48.4\) and standard deviation \(6.3\).
   
   a. Find the mean and standard deviation of \(\overline{X}\) for samples of size \(64\).
   
   b. Find the probability that the mean of a sample of size \(64\) will be less than \(46.7\).

5. A normally distributed population has mean \(25.6\) and standard deviation \(3.3\).
   
   a. Find the probability that a single randomly selected element \(X\) of the population exceeds \(30\).
   
   b. Find the mean and standard deviation of \(\overline{X}\) for samples of size \(9\).
   
   c. Find the probability that the mean of a sample of size \(9\) drawn from this population exceeds \(30\).
6. A normally distributed population has mean $57.7$ and standard deviation $12.1$.
   a. Find the probability that a single randomly selected element $X$ of the population is less than $45$.
   b. Find the mean and standard deviation of $\overline{X}$ for samples of size $16$.
   c. Find the probability that the mean of a sample of size $16$ drawn from this population is less than $45$.
7. A population has mean $557$ and standard deviation $35$.
   a. Find the mean and standard deviation of $\overline{X}$ for samples of size $50$.
   b. Find the probability that the mean of a sample of size $50$ will be more than $570$.
8. A population has mean $16$ and standard deviation $1.7$.
   a. Find the mean and standard deviation of $\overline{X}$ for samples of size $80$.
   b. Find the probability that the mean of a sample of size $80$ will be more than $16.4$.
9. A normally distributed population has mean $1,214$ and standard deviation $122$.
   a. Find the probability that a single randomly selected element $X$ of the population is between $1,100$ and $1,300$.
   b. Find the mean and standard deviation of $\overline{X}$ for samples of size $25$.
   c. Find the probability that the mean of a sample of size $25$ drawn from this population is between $1,100$ and $1,300$.
10. A normally distributed population has mean $57,800$ and standard deviation $750$.
    a. Find the probability that a single randomly selected element $X$ of the population is between $57,000$ and $58,000$.
    b. Find the mean and standard deviation of $\overline{X}$ for samples of size $100$.
    c. Find the probability that the mean of a sample of size $100$ drawn from this population is between $57,000$ and $58,000$.
11. A population has mean $72$ and standard deviation $6$.
    a. Find the mean and standard deviation of $\overline{X}$ for samples of size $45$.
    b. Find the probability that the mean of a sample of size $45$ will differ from the population mean $72$ by at least $2$ units, that is, is either less than $70$ or more than $74$. (Hint: One way to solve the problem is to first find the probability of the complementary event.)
12. A population has mean $12$ and standard deviation $1.5$.
    a. Find the mean and standard deviation of $\overline{X}$ for samples of size $90$.
    b. Find the probability that the mean of a sample of size $90$ will differ from the population mean $12$ by at least $0.3$ unit, that is, is either less than $11.7$ or more than $12.3$. (Hint: One way to solve the problem is to first find the probability of the complementary event.)

Applications

13. Suppose the mean number of days to germination of a variety of seed is $22$, with standard deviation $2.3$ days. Find the probability that the mean germination time of a sample of $160$ seeds will be within $0.5$ day of the population mean.
14. Suppose the mean length of time that a caller is placed on hold when telephoning a customer service center is $23.8$ seconds, with standard deviation $4.6$ seconds. Find the probability that the mean length of time on hold in a sample of $1,200$ calls will be within $0.5$ second of the population mean.
15. Suppose the mean amount of cholesterol in eggs labeled “large” is $186$ milligrams, with standard deviation $7$ milligrams.
milligrams. Find the probability that the mean amount of cholesterol in a sample of \(\{144\}\) eggs will be within \(\{2\}\) milligrams of the population mean.

16. Suppose that in one region of the country the mean amount of credit card debt per household in households having credit card debt is \(\{\$15,250\}\), with standard deviation \(\{\$7,125\}\). Find the probability that the mean amount of credit card debt in a sample of \(\{1,600\}\) such households will be within \(\{\$300\}\) of the population mean.

17. Suppose speeds of vehicles on a particular stretch of roadway are normally distributed with mean \(\{36.6\}\) mph and standard deviation \(\{1.7\}\) mph.
   a. Find the probability that the speed \(\{X\}\) of a randomly selected vehicle is between \(\{35\}\) and \(\{40\}\) mph.
   b. Find the probability that the mean speed \(\{\overline{X}\}\) of \(\{20\}\) randomly selected vehicles is between \(\{35\}\) and \(\{40\}\) mph.

18. Many sharks enter a state of tonic immobility when inverted. Suppose that in a particular species of sharks the time a shark remains in a state of tonic immobility when inverted is normally distributed with mean \(\{11.2\}\) minutes and standard deviation \(\{1.1\}\) minutes.
   a. If a biologist induces a state of tonic immobility in such a shark in order to study it, find the probability that the shark will remain in this state for between \(\{10\}\) and \(\{13\}\) minutes.
   b. When a biologist wishes to estimate the mean time that such sharks stay immobile by inducing tonic immobility in each of a sample of \(\{12\}\) sharks, find the probability that mean time of immobility in the sample will be between \(\{10\}\) and \(\{13\}\) minutes.

19. Suppose the mean cost across the country of a \(\{30\}\)-day supply of a generic drug is \(\{\$46.58\}\), with standard deviation \(\{\$4.84\}\). Find the probability that the mean of a sample of \(\{100\}\) prices of \(\{30\}\)-day supplies of this drug will be between \(\{\$45\}\) and \(\{\$50\}\).

20. Suppose the mean length of time between submission of a state tax return requesting a refund and the issuance of the refund is \(\{47\}\) days, with standard deviation \(\{6\}\) days. Find the probability that in a sample of \(\{50\}\) returns requesting a refund, the mean such time will be more than \(\{50\}\) days.

21. Scores on a common final exam in a large enrollment, multiple-section freshman course are normally distributed with mean \(\{72.7\}\) and standard deviation \(\{13.1\}\).
   a. Find the probability that the score \(\{X\}\) on a randomly selected exam paper is between \(\{70\}\) and \(\{80\}\).
   b. Find the probability that the mean score \(\{\overline{X}\}\) of \(\{38\}\) randomly selected exam papers is between \(\{70\}\) and \(\{80\}\).

22. Suppose the mean weight of school children’s bookbags is \(\{17.4\}\) pounds, with standard deviation \(\{2.2\}\) pounds. Find the probability that the mean weight of a sample of \(\{30\}\) bookbags will exceed \(\{17\}\) pounds.

23. Suppose that in a certain region of the country the mean duration of first marriages that end in divorce is \(\{7.8\}\) years, standard deviation \(\{1.2\}\) years. Find the probability that in a sample of \(\{75\}\) divorces, the mean age of the marriages is at most \(\{8\}\) years.

24. Borachio eats at the same fast food restaurant every day. Suppose the time \(\{X\}\) between the moment Borachio enters the restaurant and the moment he is served his food is normally distributed with mean \(\{4.2\}\) minutes and standard deviation \(\{1.3\}\) minutes.
   a. Find the probability that when he enters the restaurant today it will be at least \(\{5\}\) minutes until he is served.
   b. Find the probability that average time until he is served in eight randomly selected visits to the restaurant will be at least \(\{5\}\) minutes.

Additional Exercises

25. A high-speed packing machine can be set to deliver between \(\{11\}\) and \(\{13\}\) ounces of a liquid. For any delivery setting in this range the amount delivered is normally distributed with mean some amount \(\{\mu\}\) and with standard
deviation \(0.08\) ounce. To calibrate the machine it is set to deliver a particular amount, many containers are filled, and \(25\) containers are randomly selected and the amount they contain is measured. Find the probability that the sample mean will be within \(0.05\) ounce of the actual mean amount being delivered to all containers.

26. A tire manufacturer states that a certain type of tire has a mean lifetime of \(60,000\) miles. Suppose lifetimes are normally distributed with standard deviation \(\sigma =3,500\) miles.

   a. Find the probability that if you buy one such tire, it will last only \(57,000\) or fewer miles. If you had this experience, is it particularly strong evidence that the tire is not as good as claimed?

   b. A consumer group buys five such tires and tests them. Find the probability that average lifetime of the five tires will be \(57,000\) miles or less. If the mean is so low, is that particularly strong evidence that the tire is not as good as claimed?

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**Answers**

1. a. \(\mu_{\overline{X}}=128, \sigma_{\overline{X}}=3.67\)
   
   b. \(0.9936\)

2.

3. a. \(\mu_{\overline{X}}=73.5, \sigma_{\overline{X}}=0.456\)
   
   b. \(0.0005\)

4.

5. a. \(0.0918\)
   
   b. \(\mu_{\overline{X}}=25.6, \sigma_{\overline{X}}=1.1\)
   
   c. \(0.0000\)

6.

7. a. \(\mu_{\overline{X}}=557, \sigma_{\overline{X}}=4.9497\)
   
   b. \(0.0043\)

8.

9. a. \(0.5818\)
   
   b. \(\mu_{\overline{X}}=1214, \sigma_{\overline{X}}=24.4\)
   
   c. \(0.9998\)

10.

11. a. \(\mu_{\overline{X}}=72, \sigma_{\overline{X}}=0.8944\)
   
   b. \(0.0250\)

12.

13. \(0.9940\)

14. \(0.9994\)

15.

16.

17. a. \(0.8036\)
   
   b. \(1.0000\)

18.

19. \(0.9994\)

20.

21. a. \(0.2955\)

22. a. \(0.8036\)
   
   b. \(1.0000\)

23. \(0.9994\)

24. \(0.2955\)
The proportion of a population with a characteristic of interest is \(p = 0.37\). Find the mean and standard deviation of the sample proportion \(\hat{p}\) obtained from random samples of size \(1,600\).

2. The proportion of a population with a characteristic of interest is \(p = 0.82\). Find the mean and standard deviation of the sample proportion \(\hat{p}\) obtained from random samples of size \(900\).

3. The proportion of a population with a characteristic of interest is \(p = 0.76\). Find the mean and standard deviation of the sample proportion \(\hat{p}\) obtained from random samples of size \(1,200\).

4. The proportion of a population with a characteristic of interest is \(p = 0.37\). Find the mean and standard deviation of the sample proportion \(\hat{p}\) obtained from random samples of size \(125\).

5. Random samples of size \(225\) are drawn from a population in which the proportion with the characteristic of interest is \(0.25\). Decide whether or not the sample size is large enough to assume that the sample proportion \(\hat{p}\) is normally distributed.

6. Random samples of size \(1,600\) are drawn from a population in which the proportion with the characteristic of interest is \(0.05\). Decide whether or not the sample size is large enough to assume that the sample proportion \(\hat{p}\) is normally distributed.

7. Random samples of size \(n\) produced sample proportions \(\hat{p}\) as shown. In each case decide whether or not the sample size is large enough to assume that the sample proportion \(\hat{p}\) is normally distributed.
   a. \(n = 50; \hat{p} = 0.48\)
   b. \(n = 50; \hat{p} = 0.12\)
   c. \(n = 100; \hat{p} = 0.12\)

8. Samples of size \(n\) produced sample proportions \(\hat{p}\) as shown. In each case decide whether or not the sample size is large enough to assume that the sample proportion \(\hat{p}\) is normally distributed.
   a. \(n = 30; \hat{p} = 0.72\)
   b. \(n = 30; \hat{p} = 0.84\)
   c. \(n = 75; \hat{p} = 0.84\)

9. A random sample of size \(121\) is taken from a population in which the proportion with the characteristic of interest...
is \( p = 0.47 \). Find the indicated probabilities.

a. \( P(0.45 \leq \widehat{P} \leq 0.50) \)

b. \( P(\widehat{P} \geq 0.50) \)

10. A random sample of size \( 225 \) is taken from a population in which the proportion with the characteristic of interest is \( p = 0.34 \). Find the indicated probabilities.

a. \( P(0.25 \leq \widehat{P} \leq 0.40) \)

b. \( P(\widehat{P} \geq 0.35) \)

11. A random sample of size 900 is taken from a population in which the proportion with the characteristic of interest is \( p = 0.62 \). Find the indicated probabilities.

a. \( P(0.60 \leq \widehat{P} \leq 0.64) \)

b. \( P(0.57 \leq \widehat{P} \leq 0.67) \)

12. A random sample of size 1,100 is taken from a population in which the proportion with the characteristic of interest is \( p = 0.28 \). Find the indicated probabilities.

1. \( P(0.27 \leq \widehat{P} \leq 0.29) \)

2. \( P(0.23 \leq \widehat{P} \leq 0.33) \)

Applications

13. Suppose that \( 8\% \) of all males suffer some form of color blindness. Find the probability that in a random sample of \( 250 \) men at least \( 10\% \) will suffer some form of color blindness. First verify that the sample is sufficiently large to use the normal distribution.

14. Suppose that \( 29\% \) of all residents of a community favor annexation by a nearby municipality. Find the probability that in a random sample of \( 50 \) residents at least \( 35\% \) will favor annexation. First verify that the sample is sufficiently large to use the normal distribution.

15. Suppose that \( 2\% \) of all cell phone connections by a certain provider are dropped. Find the probability that in a random sample of \( 1,500 \) calls at most \( 40\% \) will be dropped. First verify that the sample is sufficiently large to use the normal distribution.

16. Suppose that in \( 20\% \) of all traffic accidents involving an injury, driver distraction in some form (for example, changing a radio station or texting) is a factor. Find the probability that in a random sample of \( 275 \) such accidents between \( 15\% \) and \( 25\% \) involve driver distraction in some form. First verify that the sample is sufficiently large to use the normal distribution.

17. An airline claims that \( 72\% \) of all its flights to a certain region arrive on time. In a random sample of \( 30 \) recent arrivals, \( 19 \) were on time. You may assume that the normal distribution applies.

a. Compute the sample proportion.

b. Assuming the airline’s claim is true, find the probability of a sample of size \( 30 \) producing a sample proportion so low as was observed in this sample.

18. A humane society reports that \( 19\% \) of all pet dogs were adopted from an animal shelter. Assuming the truth of this assertion, find the probability that in a random sample of \( 80 \) pet dogs, between \( 15\% \) and \( 20\% \) were adopted from a shelter. You may assume that the normal distribution applies.

19. In one study it was found that \( 86\% \) of all homes have a functional smoke detector. Suppose this proportion is valid for all homes. Find the probability that in a random sample of \( 600 \) homes, between \( 80\% \) and \( 90\% \) will have a functional smoke detector. You may assume that the normal distribution applies.

20. A state insurance commission estimates that \( 13\% \) of all motorists in its state are uninsured. Suppose this
21. An outside financial auditor has observed that about \(4\%)\) of all documents he examines contain an error of some sort. Assuming this proportion to be accurate, find the probability that a random sample of \(700\) documents will contain at least \(30\) with some sort of error. You may assume that the normal distribution applies.

22. Suppose \(7\%)\) of all households have no home telephone but depend completely on cell phones. Find the probability that in a random sample of \(450\) households, between \(25\) and \(35\) will have no home telephone. You may assume that the normal distribution applies.

Additional Exercises

23. Some countries allow individual packages of prepackaged goods to weigh less than what is stated on the package, subject to certain conditions, such as the average of all packages being the stated weight or greater. Suppose that one requirement is that at most \(4\%)\) of all packages marked \(500\) grams can weigh less than \(490\) grams. Assuming that a product actually meets this requirement, find the probability that in a random sample of \(150\) such packages the proportion weighing less than \(490\) grams is at least \(3\%)\). You may assume that the normal distribution applies.

24. An economist wishes to investigate whether people are keeping cars longer now than in the past. He knows that five years ago, \(38\%)\) of all passenger vehicles in operation were at least ten years old. He commissions a study in which \(325\) automobiles are randomly sampled. Of them, \(132\) are ten years old or older.
   a. Find the sample proportion.
   b. Find the probability that, when a sample of size \(325\) is drawn from a population in which the true proportion is \(0.38\%), the sample proportion will be as large as the value you computed in part (a). You may assume that the normal distribution applies.
   c. Give an interpretation of the result in part (b). Is there strong evidence that people are keeping their cars longer than was the case five years ago?

25. A state public health department wishes to investigate the effectiveness of a campaign against smoking. Historically \(22\%)\) of all adults in the state regularly smoked cigars or cigarettes. In a survey commissioned by the public health department, \(279\) of \(1,500\) randomly selected adults stated that they smoke regularly.
   a. Find the sample proportion.
   b. Find the probability that, when a sample of size \(1,500\) is drawn from a population in which the true proportion is \(0.22\), the sample proportion will be no larger than the value you computed in part (a). You may assume that the normal distribution applies.
   c. Give an interpretation of the result in part (b). How strong is the evidence that the campaign to reduce smoking has been effective?

26. In an effort to reduce the population of unwanted cats and dogs, a group of veterinarians set up a low-cost spay/neuter clinic. At the inception of the clinic a survey of pet owners indicated that \(78\%)\) of all pet dogs and cats in the community were spayed or neutered. After the low-cost clinic had been in operation for three years, that figure had risen to \(86\%)\).
   a. What information is missing that you would need to compute the probability that a sample drawn from a population in which the proportion is \(78\%)\) (corresponding to the assumption that the low-cost clinic had had no effect) is as high as \(86\%)\)?
   b. Knowing that the size of the original sample three years ago was \(150\) and that the size of the recent sample was \(125\), compute the probability mentioned in part (a). You may assume that the normal distribution applies.
   c. Give an interpretation of the result in part (b). How strong is the evidence that the presence of the low-cost
has increased the proportion of pet dogs and cats that have been spayed or neutered?

27. An ordinary die is “fair” or “balanced” if each face has an equal chance of landing on top when the die is rolled. Thus the proportion of times a three is observed in a large number of tosses is expected to be close to \((1/6)\) or \((0.1\overline{6})\). Suppose a die is rolled \((240)\) times and shows three on top \((36)\) times, for a sample proportion of \((0.15)\).

a. Find the probability that a fair die would produce a proportion of \((0.15)\) or less. You may assume that the normal distribution applies.

b. Give an interpretation of the result in part (b). How strong is the evidence that the die is not fair?

c. Suppose the sample proportion \((0.15)\) came from rolling the die \((2,400)\) times instead of only \((240)\) times. Rework part (a) under these circumstances.

d. Give an interpretation of the result in part (c). How strong is the evidence that the die is not fair?

Answers

1. \(\mu _{\widehat{P}}=0.37, \sigma _{\widehat{P}}=0.012\)

2. 

3. \(\mu _{\widehat{P}}=0.76, \sigma _{\widehat{P}}=0.012\)

4. 

5. \((p\pm 3\sqrt{\frac{pq}{n}}=0.25\pm 0.087, \text{yes})\)

6. 

7. a. \((\hat{p}\pm 3\sqrt{\frac{\hat{p}\hat{q}}{n}}=0.48\pm 0.21, \text{yes})\)

b. \((\hat{p}\pm 3\sqrt{\frac{\hat{p}\hat{q}}{n}}=0.12\pm 0.14, \text{no})\)

c. \((\hat{p}\pm 3\sqrt{\frac{\hat{p}\hat{q}}{n}}=0.12\pm 0.10, \text{yes})\)

8. 

9. a. \((0.4154)\)

b. \((0.2546)\)

10. 

11. a. \((0.7850)\)

b. \((0.9980)\)

12. 

13. \((p\pm 3\sqrt{\frac{pq}{n}}=0.08\pm 0.05)\) and \([0.03,0.13]\subset [0,1],0.1210\)

14. 

15. \((p\pm 3\sqrt{\frac{pq}{n}}=0.02\pm 0.01)\) and \([0.01,0.03]\subset [0,1],0.9671\)

16. 

17. a. \((0.63)\)

b. \((0.1446)\)

18. 

19. \((0.9977)\)

20. 

21. \((0.3483)\)

22. 

23. \((0.7357)\)
25.  a. \((0.186)\)
    b. \((0.0007)\)
    c. In a population in which the true proportion is \((22\%)\) the chance that a random sample of size \((1,500)\) would produce a sample proportion of \((18.6\%)\) or less is only \((7/100)\) of \((1\%)\). This is strong evidence that currently a smaller proportion than \((22\%)\) smoke.

26.  

27.  a. \((0.2451)\)
    b. We would expect a sample proportion of \((0.15)\) or less in about \((24.5\%)\) of all samples of size \((240)\), so this is practically no evidence at all that the die is not fair.
    c. \((0.0139)\)
    d. We would expect a sample proportion of \((0.15)\) or less in only about \((1.4\%)\) of all samples of size \((2,400)\), so this is strong evidence that the die is not fair.

**Contributor**

- Anonymous