

Statistics Ch 10 Hypothesis Test for 2 samples

Ch 10.3 Hypothesis Test for 2 proportions.

Main objectives in chapter 10

- Compare the same population parameter (p or μ) from two different populations by using sample from each of the two populations.
- Use the samples to tell if the population parameters are significantly different, are the same, or one is higher (or lower) than the other.
- Use the samples to estimate the difference in the two population parameters if there is significant difference.

Terms:

Significant different – means the parameters are not the same statistically, taking into considering of sampling error.

Independent vs dependent samples

Samples are independent if values are not naturally paired.

Inference about proportions from two populations.

- 1) Test a claim about p_1 and p_2 where p_1 and p_2 are population proportion (of the same category) from population one and two.
- 2) Estimate the confidence interval of the difference of $p_1 - p_2$.

Notations:

$\hat{p}_1 = \frac{x_1}{n_1}$, $\hat{p}_2 = \frac{x_2}{n_2}$. These are sample proportions from the two populations. ($x_1 = n_1 \cdot \hat{p}_1$, $x_2 = n_2 \cdot \hat{p}_2$)

Pooled sample: (variances are pooled)

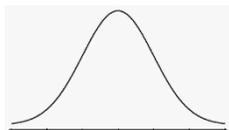
Assume p_1 and p_2 are the same, we can combine the two samples to create a pooled sample $\bar{p} = \frac{x_1 + x_2}{n_1 + n_2}$;

$$\bar{q} = 1 - \bar{p}$$

The difference of two sample proportions $\hat{p}_1 - \hat{p}_2$ will be normally distributed with

mean = $p_1 - p_2$, $sd = \sqrt{\frac{\bar{p}\bar{q}}{n_1} + \frac{\bar{p}\bar{q}}{n_2}}$ where

$x_1, x_2, n_1 - x_1, n_2 - x_2$ are at least 5.



Steps for Testing claim of 2 population proportions:

- 1) Define populations, record n_1, x_1, n_2, x_2 , Requirement is $x_1, n_1 - x_1, x_2, n_2 - x_2$ are at least 5.
- 2) Record the claim in symbolic form:
 p_1 ($=, <, >$ or \neq) p_2 .
Write H_0 ($p_1 = p_2$) and H_a ($p_1 <, >$ or $\neq p_2$) in symbolic form.
Note: p_1 must be on the left, p_2 on the right.
- 3) Identify significant level, type of test (left-tail, right-tail or two-tail test) and the sampling distribution.
For proportion, distribution is z-normal.
- 4) Use **Statdisk/Hypothesis/ proportion Two samples** to find test statistic and p-value.
 - Select H_a : $p_1 \neq p_2$ or $p_1 > p_2$ or $p_1 < p_2$.
 - Enter significant level: α
 - Enter x_1, n_1, x_2, n_2 . Evaluate.Output:
 - test statistic, z – describe how many SD the sample difference $\hat{p}_1 - \hat{p}_2$ is from the H_0 assumption of $p_1 = p_2$, ($p_1 - p_2 = 0$) in a normal distribution.
 - P-value: probability of getting the sample difference $\hat{p}_1 - \hat{p}_2$ or worse if $p_1 = p_2$ is true.
 - Confidence interval at appropriate C-level
lower bound $< p_1 - p_2 <$ upper bound
- 5) If $P\text{-value} \leq \alpha$, reject H_0 , conclude there is significant difference between p_1 and p_2 .
If $p\text{-value} > \alpha$, fail to reject H_0 , conclude there is no significant difference between p_1 and p_2 . The difference $\hat{p}_1 - \hat{p}_2$ are due to sample variation.
- 6) Write conclusion about the claim:
There is (sufficient / not sufficient) evidence to (reject / support) the claim.
Use “sufficient evidence” if sample is significant.
Use “reject” if claim is H_0 , use “support” if claim is H_a .
Or use Claim conclusion flowchart.
- 7) Make conclusion using Confidence Interval:
 - a) Confidence interval is included in an hypothesis test output. OR
 - b) Use Statdisk/Confidence Interval/Proportion 2 sample to find confidence interval.Input appropriate C-level:
two-tail test: $C\text{-level} = 1 - \alpha$.
Left-tail or right-tail test: $C\text{-level} = 1 - 2\alpha$

input n_1, x_1, n_2, x_2 .

Output: lower limit $< p_1 - p_2 <$ upper limit.

Make conclusion below:

- i) If the interval contains zero or both positive and negative values, conclude p_1 and p_2 has no significant difference. Sample is not significant.
(L limit is negative U limit is positive.)
- ii) If the interval is all positive, conclude $p_1 > p_2$
(L. Limit and U. Limits are both positive.)
- iii) If the interval is all negative, conclude $p_1 < p_2$
(L. limit and U. Limits are both negative.)

Note: appropriate C-level:

two-tail test: C-level = $1 - \alpha$.

Left-tail or right-tail test: C-level = $1 - 2\alpha$

Summary:

$p_1 - p_2 > 0$ implies $p_1 > p_2$

$p_1 - p_2 < 0$ implies $p_1 < p_2$

$p_1 - p_2 = 0$ implies $p_1 = p_2$

Note: Conclusion from hypothesis test is more accurate than conclusion from Confidence Interval for Proportion test. Check p-value to confirm if sample difference is significant.

p-value $\leq \alpha$ there is significant difference.

p-value $> \alpha$, there is no significant difference.

Ex1: Given the table below about number of cars with Rare and Front License plates at two states.

	Connecticut	New York
Cars with rare license plates only	239	9
cars with both front and rare license plates	1810	541
Total	2049	550

The sample proportion for Connecticut is $239/2049$ (11.7%) The sample proportion for New York = $9/550$ (1.6%). The samples are different, but are they statistically different?

a) Test with a significant level of 0.05 that the proportions of car with only rare license plate in Connecticut and New York are the same.

1) Define population 1 = Connecticut, , $n_1 = 2049, x_1 = 239$, population 2 = New York, $n_2 = 550, x_2 = 9$.

2) Claim: $p_1 = p_2$, $H_0: p_1 = p_2$, $H_a: p_1 \neq p_2$

3) Significant level = 0.05, H_a use \neq , so two-tail test, distribution is z.

4) Use Statdisk/Hypothesis/proportion 2 samples, Select $H_a: p_1 \neq p_2$. significance = 0.05, input $n_1 = 2049, x_1 = 239, n_2 = 550, x_2 = 9$. Evaluate.

Test statistic $z = 7.11$, p-value = 0.0000

5) Since p-value (0.0000) $< \alpha$ (0.05), reject H_0

6) Since H_0 is the claim and we rejected H_0 , use Box1 in flow chart.

Conclude there is sufficient evidence to reject the claim that the proportion of cars with rare only License plate are the same with that of New York.

b) The 95% confidence Interval of the difference is $0.083 < p_1 - p_2 < 0.118$

Conclude the difference is between 8.3% to 11.8% at a confidence level of 95%.

Since the interval contains all positive values, the two proportions are significantly different and not the same as stated in the claim.

Note: when x is not given: $x = p n$

when n is not given: $n = \text{success} + \text{failure}$.

Ex2: An experiment is set up to test the effectiveness of Aspirin in preventing heart disease. 11,037 adults were treated with aspirin and another 11,034 adults were given placebo. Among the subjects in the treatment group, 1.26% experienced heart attacks. Among the subjects given placebos, 2.17% experienced heart attacks.

a) Use a 0.05 significant level to test the claim that aspirin is effective in lowering heart attacks.

i) Def population 1 – treatment group, $n_1 = 11037, x_1 = 0.0126(11037) = 139$.

population 2 – placebo group, $n_2 = 11034, x_2 = 0.0217(11034) = 239$.

ii) Claim $p_1 < p_2$, $H_0: p_1 = p_2$, $H_a: p_1 < p_2$

iii) $\alpha = 0.05$, H_a is " $<$ ", so left-tail test, use z-distribution

iv) Use Statdisk/hypothesis/Proportion 2 samples, select $H_a: p_1 < p_2$, significance = 0.05, enter n_1, x_1, n_2, x_2 , evaluate.

Output: test statistic $z = -5.19$, p-value = 0.0000

$-0.012 < p_1 - p_2 < -0.006$

v) p-value (0.0000) < 0.05 Reject H_0 , there is significant difference between p_1 and p_2 .

vi) Since claim is H_1 , we use support statement, Since we reject H_0 , we use "sufficient evidence". Flow chart will be at box 3.

There is sufficient evidence that to support the claim that aspirin is effective in lowering heart attacks.

b) The 90% confidence interval is calculated because the test is a left-tail test. C-level = $1 - 2(0.05) = 0.90$

$-0.012 < p_1 - p_2 < -0.006$

Since confidence interval does not contain zero, conclude there is significant difference.

Since the whole interval is negative, conclude $p_1 < p_2$.

implying aspirin can lower heart attack.

c) Based on the result, would aspirin be recommended for adults to avoid heart attacks?

Yes, the result conclude that aspirin is effective.

d) Would the result be useful if the treatment group are selected from males only and the placebo group are selected from females only.

The result may not apply to general population because the two groups are not only different on the treatment. The result may be due to other confounding factors between male and female.

Ex3. In a randomly picked year from 1985 to present, there were 1840 Hispanic students at Cabrillo College out of a total of 12328 students. At Lake Tahoe College, there were 321 Hispanic students out of a total of 2441 students. Test the claim at a significant level of 1% that the percent of Hispanic student is higher in Cabrillo College.

Ans:

1) Define population 1 – Cabrillo College $n_1 = 12328$, $x_1 = 1840$, population 2 – Lake Tahoe College, $n_2 = 2441$, $x_2 = 321$.

Ex4. In a random sample of 100 forests in the United States, 56 were coniferous or contained conifers. In a random sample of 80 forests in Mexico, 40 were coniferous. Is the proportion of conifers in the United States statistically more than the proportion of conifers in Mexico? At a significant level of 1%, construct an appropriate confidence interval to test the claim and estimate the difference.

Ans:

Define population1 -United States forests, $n_1 = 100$, $x_1 = 56$.

Population 2 – Mexico's forests, $n_2 = 80$, $x_2 = 40$

Claim: $p_1 > p_2$. $H_0: p_1 = p_2$, $H_a: p_1 > p_2$ (right tail test)

We can do a confidence interval to determine if $p_1 > p_2$.

Appropriate C-level = $1 - 2\alpha$ (right-tail test) = 0.98

statdisk/Confidence Interval/proportion 2 samples/
input Clevel = 0.98 ($1 - 2(0.01)$)

Input $n_1 = 100$, $x_1 = 56$, $x_2 = 80$, $n_2 = 40$, evaluate

Output: $-0.114 < p_1 - p_2 < 0.234$

Since 0 is in the interval, conclude there is no statistical difference between p_1 and p_2 . $p_1 = p_2$, Sample is not significant. (use “not sufficient evidence” statement). H_a is claim. (use “support”) Conclusion: There is not sufficient evidence to support the claim that the proportion of conifers in the United States statistically more than the proportion of conifers in Mexico. The difference in percentage can be between -11.4% to 23.4% at 98% confidence.

Ch 10.1 and 10.4 Hypothesis Test for 2 population means

Independent and Dependent samples

Independent samples: - sample values from one population are not related to or naturally paired or matched with the sample values from the other.

Sample size can be same or different.

Summary statistics or data can be given.

Dependent samples: - sample values are some how matched, where the matching is based on some inherent relationship. (This can be measurement from the same subject before/after, or each pair consists of matched pairs such as husband/wife, twin of siblings.)

Note: “match paired” does not mean cause/effect.

Sample size must be the same. Paired sample data will be given.

Note: For experiment to compare two populations, dependent samples results are more favorable.

Ex1. sample 1: heights of 14 men.

sample 2: heights of 16 women.

The above samples are independent.

Ex2: sample 1: Heights of husband of a couple.

sample 2: Heights of the wife of each couple.

Since each paired value are from the same couple. The samples are dependent.

Use independent samples to compare population means

To compare population mean (μ_1 and μ_2) from two populations, sample means (\bar{x}_1 and \bar{x}_2) are collected. If \bar{x}_1 and \bar{x}_2 are normally distributed, then the difference $\bar{x}_1 - \bar{x}_2$ will be also be normally distributed.

The standard deviation of $\bar{x}_1 - \bar{x}_2$ will have a value that is square root of the sum of variances.

The most common case is both σ_1 and σ_2 are unknown and unequal.

Hence variances **are not pooled**, and sample variances are used to standardize the distribution of

$$\bar{x}_1 - \bar{x}_2 \text{ with } t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$$df = \frac{\left(\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}\right)^2}{\left(\frac{1}{n_1-1}\right)\left(\frac{(s_1)^2}{n_1}\right)^2 + \left(\frac{1}{n_2-1}\right)\left(\frac{(s_2)^2}{n_2}\right)^2}$$



Note: by default, when σ_1 and σ_2 are not given, we assume σ_1 and σ_2 are unknown and unequal, resulting in not pooling the variances.

Use paired difference (d) to compare population means.

When samples are dependent, we define a new variable $d = x_1 - x_2$ for each paired data.

The distribution of \bar{d} will be normal if X_1 and X_2 are normal or sample size $n_1 = n_2 = n > 30$. The sample mean of d is \bar{d} with standard deviation s_d .

The distribution of \bar{d} will have a standard deviation s_d / \sqrt{n} . Hence t distribution is used to describe the standard error with $t = \frac{\bar{d} - \mu_d}{s_d / \sqrt{n}}$

Note: We are not comparing $\bar{x}_1 - \bar{x}_2$ but analyzing a new variable $d = (x_1 - x_2)$ from each paired sample.

Note: statdisk does not give a new column of d in the output but provide a mean of d and sd of d .

Inference about means from two populations.

a) Test a claim about μ_1 and μ_2 where μ_1 and μ_2 are population mean (of the same type of measurement) from population one and two.

b) Estimate the confidence interval of the difference of $\mu_1 - \mu_2$ or mean differences (μ_d).

Note: The procedure works only if $n_1, n_2 > 30$ or X_1, X_2 are normal. Assume σ_1 and σ_2 are unknown and unequal so no pooling of variances is used.

Steps:

1) Determine if the two samples are independent or dependent.

If samples are independent,

Define population 1, record n_1, \bar{x}_1, s_1 .

Define population 2, record n_2, \bar{x}_2, s_2

If data are given, enter data to columns in Statdisk.

If samples are dependent,

Define the *difference* $d = (x_1 - x_2)$.

Enter data to 2 columns.

2) Set up Claim, H_0 and H_1 according to claim statement.

For independent samples:

Claim: $\mu_1 (=, <, > \text{ or } \neq) \mu_2, H_0: \mu_1 = \mu_2,$

$H_a: \mu_1 (<, > \text{ or } \neq) \mu_2$

For dependent samples:

Claim: $\mu_d (=, <, > \text{ or } \neq) 0, H_0: \mu_d = 0,$

$H_a: \mu_d (<, > \text{ or } \neq) 0$

3) Identify significant level and type of test. distribution is t and not pooling variances.

4) For independent sample – use

statdisk/Analysis/Hypothesis/ Mean 2 independent samples. Select alternative Hypothesis and significant level.

If statistic summaries are given, enter $n_1, \bar{x}_1, s_1,$ and n_2, \bar{x}_2, s_2 . If data are given, click “use data tab”, select columns containing sample 1 and 2.

Use the default “unequal variances, no pooled”.

Evaluate.

Output: test statistic (t) and p-value (p)

confidence interval of $\mu_1 - \mu_2$ at

appropriate level of $1-\alpha$ or $1-2\alpha$

For dependent sample –

statdisk/Analysis/Hypothesis/ Mean Matched pairs.

Select alternative Hypothesis and significant level.

Select the column where matched-pair samples are inputted.

Evaluate.

Output: test statistic (t) and p-value (p)

confidence interval of μ_d

- 5) If $P\text{-value} \leq \alpha$, reject H_0 , conclude there is significant difference between μ_1 and μ_2 .
If $p\text{-value} > \alpha$, fail to reject H_0 , conclude there is no significant difference between μ_1 and μ_2 .
- 6) Write conclusion about the claim. Use table or flowchart.
- 7) If there is significant difference, use the confidence interval for the difference $\mu_1 - \mu_2$ or μ_d where $C\text{-level}$ is $1 - \alpha$ (2-tail test) or $1 - 2\alpha$ (1-tail test)

8 Optional)

Use Confidence Interval With any C-level to test a claim or estimate mean differences $\mu_1 - \mu_2$ or mean differences (μ_d).

For independent samples: Use Statdisk/Analysis / Confidence intervals/Mean two independent samples/

For dependent samples: use Statdisk/Analysis/ Confidence intervals/Mean Match pairs

Enter all inputs or data columns:

Output

Confidence interval: L. limit $< \mu_1 - \mu_2 < U.$ limit, or
L. limit $< \mu_d < U.$ limit

p-value:

9) Make conclusion using C-level:

- If the interval contains zero, conclude μ_1 and μ_2 have no significant difference. (L limit is negative U limit is positive.)
- If the interval is all positive, conclude $\mu_1 > \mu_2$. (L. Limit and U. Limits are both positive.)
- If the interval is all negative, conclude $\mu_1 < \mu_2$. (L. limit and U. Limits are both negative.)

Note: When H_0 is rejected, there is sufficient evidence to conclude that there is significant difference between μ_1 and μ_2 .

Note: Conclusion from hypothesis test is exactly the same as the conclusion from Confidence Interval for testing of the mean because the standard errors are the same.

Ex1. A study claims that mean enrollment at two-year college is lower than at four-year colleges in the United States. Two samples are collected, from the 35 two-year colleges surveyed, the mean enrollment was 5,068 with a standard deviation of 4,777. Of the 35 four-year colleges surveyed, the mean enrollment was 5,466 with a standard deviation of 8,191. Test at sign. level of 0.05.

The samples are independent.

- Define population 1 = two-year colleges, $n_1 = 35$, $\bar{x}_1 = 5068$, $s_1 = 4777$, $\sigma_1 =$ not given,
Define population 2 = four-year colleges, $n_2 = 35$, $\bar{x}_2 = 5466$, $s_2 = 8191$, $\sigma_2 =$ not given.
- Claim: $\mu_1 < \mu_2$; $H_0 : \mu_1 = \mu_2$; $H_a = \mu_1 < \mu_2$
- Sign level = 0.05, left-tail test, use t-distribution.
- Use Statdisk/Analysis/Mean 2 independent samples
Select Alt. hypothesis : Pop Mean 1 < Pop Mean 2
Enter 0.05 to significance,
Enter $n_1 = 35$, $\bar{x}_1 = 5068$, $s_1 = 4777$, $\sigma_1 =$ not given, $n_2 = 35$, $\bar{x}_2 = 5466$, $s_2 = 8191$, $\sigma_2 =$ not given.
Use "Unequal variance, no Pool". Evaluate.
Output test stat $t = -0.248$, $p\text{-value} = 0.4024$
90% confidence interval = $-3079.7 < \mu_1 - \mu_2 < 2283.7$
- since $p\text{-value} (0.4024) > 0.05$, fail to reject H_0
- Since sample is not significant, claim is H_a .
There is not suff. evidence to support the claim the mean enrollment in two-year college is less than those at four-year college.

Ex2. Use confidence interval at 95% confidence to estimate the difference in mean life of Duracell and Eveready battery. Sample results are as follow:

Duracell: $n_1 = 8$, $\bar{x}_1 = 41$ hr, $s_1 = 18$ hr

Eveready: $n_2 = 10$, $\bar{x}_2 = 45$ hr, $s_2 = 20$ hr

i) Samples are independent.

Define population 1 – Duracell

Define population 2 – Eveready

ii) Use Statdisk/Analysis/Confidence Intervals/ Mean 2 independent samples

Enter C-level = 0.95,

Enter sample 1 : $n_1 = 8$, $\bar{x}_1 = 41$, $s_1 = 18$ hr

Enter sample 2: $n_2 = 10$, $\bar{x}_2 = 45$ hr, $s_2 = 20$ hr

Use 'Unequal variances, no Pool'. Evaluate

Output: $p\text{-value} = 0.6618$, 95% confidence interval $-23.05 < \mu_1 - \mu_2 < 15.05$ hr

iii) Since zero contains in the interval, there is no suff. evidence conclude there is significant difference.
Conclude that the mean life for the two batteries has no significant difference.

Ex3: A study of seat belt use involved children who were hospitalized after car accidents. For 123 children who were wearing seat belts, the number of days in ICU has a mean of 0.83 days and a standard deviation of 1.77 days. For the sample of 290 children who were not wearing seat belts, the number of days in ICU has a mean of 1.39 days and a standard deviation of 3.06

days. Test the claim at $\alpha = 0.05$ by hypothesis and Confidence interval that seat belt use for children is effective in lowering the degree of injuries.

i) The two samples are independent. Values are summarized.

Define population 1: children who wear seat belts
 $n_1 = 123, \bar{x}_1 = 0.83, s_1 = 1.77$

Define population 2: children who do not wear seat belts.
 $n_2 = 290, \bar{x}_2 = 1.39, s_2 = 3.06$

ii) Claim: $\mu_1 < \mu_2$, $H_0: \mu_1 = \mu_2$, $H_a: \mu_1 < \mu_2$

iii) Use $\alpha = 0.05$, left-tail test, use t distribution.

iv) Use Statdisk/Analysis/Hypothesis Testing/Mean 2 independent samples/

Select alternative hypothesis Mean 1 < Mean 2

Enter significance = 0.05,

Enter $n_1, \bar{x}_1, s_1; n_2, \bar{x}_2, s_2$

Unequal variances, no pool.

Output: $t = -2.33$ p-value = 0.0102.

90% confidence interval is $-0.9563 < \mu_1 - \mu_2 < -0.1637$

v) P-value (0.0102) < 0.05 Reject H_0 . There is significant difference between mean ICU times.

vi) There is sufficient evidence to support the claim that seat belt use for children is effective in lowering the degree of injuries.

Confidence interval is $-0.96 < \mu_1 - \mu_2 < -0.16$ days

Since interval contains negative number, conclude there is sufficient evidence to show that $\mu_1 < \mu_2$, so conclude degree of injuries for children wearing seat belts are lower.

Conclusion from hypothesis test and confidence interval are the same.

Ex4: Listed below are course evaluation scores for courses taught by female professors and male professors.

a) Use $\alpha = 0.05$ to test the claim that there is a difference in mean evaluation scores of course taught by female professors and male professors.

Female	4.3	4.3	4.4	4.0	3.4	4.7	2.9	4.0	4.3	3.4	3.4	3.3			
Male	4.5	3.7	4.2	3.9	3.1	4.0	3.8	3.4	4.5	3.8	4.3	4.4	4.1	4.2	4.0

i) The samples are independent because pairs of value are not matched or related.

Define population 1 – female professor scores

population 2 – male professor scores

Enter data to Statdisk.

ii) Claim: $\mu_1 \neq \mu_2$, $H_0: \mu_1 = \mu_2$, $H_a: \mu_1 \neq \mu_2$.

iii) Use $\alpha = 0.05$, Two-tail test, use t distribution.

iv) Use Statdisk/Hypothesis Testing/Mean 2 independent samples / Click “use data” tab

Select “Population mean 1 not = pop mean 2” as alternative hypothesis, select columns for data, sample 1 is female professors’ scores, sample 2 is male professors’ scores.

Use Unequal variances, No Pooled. Evaluate.

Output: $t = -0.66$, p-value(p) = 0.5172, $df=19.06$

Confidence interval (95%) $-0.53 < \mu_1 - \mu_2 < 0.27$.

v) p-value (0.5172) > 0.05, fail to reject H_0 . Conclude no significant difference.

vi) Since the claim is in H_1 : Conclusion about the claim: There is not sufficient evidence to support the claim that there is a difference in evaluation scores between course taught by female and male professor. Conclude plausibly no difference in scores.

vii) The confidence interval $-0.53 < \mu_1 - \mu_2 < 0.27$, contains zero, conclude there could be no significant difference between the two means. (Not sufficient evidence to support the claim of difference.

Ex5: Assume Freshmen year for college student is from September to April of the following year. Use the sample data given below with a 0.05 significance level to test the claim that there is no difference (ie the same) in mean weight change from September to April for students in their Freshman year. Samples are SRS and weights are normally distributed.

student	A	B	C	D	E
September	67	53	64	71	70
April	66	52	68	69	71

i) The samples are dependent because pairs of value are from the same student.

define $d = \text{September weight} - \text{April weight}$

Enter data to Statdisk.

ii) Claim: $\mu_d = 0$, $H_0: \mu_d = 0$, $H_1: \mu_d \neq 0$ (claim is H_0)

iii) $\alpha = 0.05$, two-tail test, use t -distribution.

iv) Use Statdisk/Analysis/Hypothesis Testings/Mean match pairs.

Select alt. hypothesis: mean of difference not = 0

Enter significance = 0.05,

Select sample 1 for September weights,

Select Sample 2 April’s weights. Evaluate

Output $t = -0.19$, p-value(p) = 0.8605

95% confidence interval is $-3.16 < \mu_d < 2.76$ kg

v) P-value(0.8605) > 0.05, fail to reject H_0 .

vi) There is not sufficient evidence to reject the claim that there is no mean weight change from September to April for students in their freshmen year.

Conclude the claim is plausibly true.

vii) Since Confidence interval is $-3.16 \text{ kg} < \mu_d < 2.76 \text{ kg}$. Interval contains zero. We are 95% confidence that the mean change in weight from September to April

can be between an increase of 3.16 kg to a decrease of 2.76 kg.

Since the difference contains zero, conclude there is no mean difference at September and April's weights.

Note: Conclusion from Hypothesis test and Confidence Interval are the same.

Ex 6: A study was conducted to investigate the effectiveness of hypnotism in reducing pain. Randomly selected subjects are given hypnotic treatment and their pain level before and after measured and recorded below. Higher level corresponds to greater level of pain.

- a) Conduct a hypothesis test to test the claim that hypnotism treatment can reduce pain. Use $\alpha = 0.05$.
- b) Construct an appropriate confidence interval for the mean difference of pain before and after hypnotism treatment.

Patients	A	B	C	D	E	F	G	H
before	6.6	6.5	9	10.3	11.3	8.1	6.3	11.6
After	6.8	2.4	7.4	8.5	8.1	6.1	3.4	2

i) The samples are dependent because each pair of values are from the same patient.

define $d = \text{before} - \text{after}$ pain level.

($d > 0$ if hypnotic treatment reduce pain)

Enter before and after data to Statdisk.

ii) Claim: $\mu_d > 0$, $H_0: \mu_d = 0$, $H_1: \mu_d > 0$

iii) $\alpha = 0.05$, two-tail test, use t -distribution

iv) Use Statdisk/Analysis/Hypothesis Testing/Mean match pair.

Select alt. hypothesis : Mean of differences > 0

Enter significance 0.05.

Select before data for sample 1,

Select after data for sample 2. Evaluate

$\bar{d} = 3.125$, $sd = 2.91$

Output $t = 3.04$ and $p\text{-value} = 0.0095$

90% confidence interval is $1.1748 < \mu_d < 5.0752$

v) $P\text{-value} (0.0095) < 0.05$ reject H_0 .

vi) There is sufficient evidence to support the claim that hypnotism treatment can reduce pain level.

vii) 90% Confidence interval is $1.17 < \mu_d < 5.08$

Since the interval does not contain zero, there is a significant difference. The interval contains only positive values. So pain level before $>$ pain level after. Conclude hypnotism treatment can reduce pain.

Note: Conclusion from Hypothesis test and Confidence interval are the same.